[2] What is fuzzy logic?

Date: 15-APR-93

Fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth -- truth values between "completely true" and "completely false". It was introduced by Dr. Lotfi Zadeh of UC/Berkeley in the 1960's as a means to model the uncertainty of natural language. (Note: Lotfi, not Lofti, is the correct spelling of his name.)

Zadeh says that rather than regarding fuzzy theory as a single theory, we should regard the process of ``fuzzification'' as a methodology to generalize ANY specific theory from a crisp (discrete) to a continuous (fuzzy) form (see "extension principle" in [2]). Thus recently researchers have also introduced "fuzzy calculus", "fuzzy differential equations", and so on (see [7]).

Fuzzy Subsets:

Just as there is a strong relationship between Boolean logic and the concept of a subset, there is a similar strong relationship between fuzzy logic and fuzzy subset theory.

In classical set theory, a subset U of a set S can be defined as a mapping from the elements of S to the elements of the set {0, 1},

```
   U: S --> {0, 1}
```

This mapping may be represented as a set of ordered pairs, with exactly one ordered pair present for each element of S. The first element of the ordered pair is an element of the set S, and the second element is an element of the set {0, 1}. The value zero is used to represent non-membership, and the value one is used to represent membership. The truth or falsity of the statement

```
    x is in U
```

is determined by finding the ordered pair whose first element is x. The statement is true if the second element of the ordered pair is 1, and the statement is false if it is 0.

Similarly, a fuzzy subset F of a set S can be defined as a set of ordered pairs, each with the first element from S, and the second element from the interval [0,1], with exactly one ordered pair present for each element of S. This defines a mapping between elements of the set S and values in the interval [0,1]. The value zero is used to represent complete non-membership, the value one is used to represent complete membership, and values in between are used to represent intermediate DEGREES OF MEMBERSHIP. The set S is referred to as the UNIVERSE OF DISCOURSE for the fuzzy subset F. Frequently, the mapping is described as a function, the MEMBERSHIP FUNCTION of F. The degree to which the statement

```
    x is in F
```

is true is determined by finding the ordered pair whose first element is x. The DEGREE OF TRUTH of the statement is the second element of the

ordered pair.

In practice, the terms "membership function" and fuzzy subset get used interchangeably.

That's a lot of mathematical baggage, so here's an example. Let's talk about people and "tallness". In this case the set S (the universe of discourse) is the set of people. Let's define a fuzzy subset TALL, which will answer the question "to what degree is person x tall?" Zadeh describes TALL as a LINGUISTIC VARIABLE, which represents our cognitive category of "tallness". To each person in the universe of discourse, we have to assign a degree of membership in the fuzzy subset TALL. The easiest way to do this is with a membership function based on the person's height.

```
    tall(x) = { 0,                       if height(x) < 5 ft.,
                (height(x)-5ft.)/2ft.,   if 5 ft. <= height (x) <= 7 ft.,
                1,                       if height(x) > 7 ft. }
```

A graph of this looks like:

```
1.0 +                  +-------------------
    |                 /
    |                /
0.5 +               /
    |              /
    |             /
0.0 +-------------+-----+-------------------
                  |     |
                 5.0   7.0

                height, ft. ->
```

Given this definition, here are some example values:

Person	Height	degree of tallness
Billy	3' 2"	0.00 [I think]
Yoke	5' 5"	0.21
Drew	5' 9"	0.38
Erik	5' 10"	0.42
Mark	6' 1"	0.54
Kareem	7' 2"	1.00 [depends on who you ask]

Expressions like "A is X" can be interpreted as degrees of truth, e.g., "Drew is TALL" = 0.38.

Note: Membership functions used in most applications almost never have as simple a shape as tall(x). At minimum, they tend to be triangles pointing up, and they can be much more complex than that. Also, the discussion characterizes membership functions as if they always are based on a single criterion, but this isn't always the case, although it is quite common. One could, for example, want to have the membership function for TALL depend on both a person's height and their age (he's tall for his age). This is perfectly legitimate, and occasionally used in practice. It's referred to as a two-dimensional membership function, or a "fuzzy relation". It's also possible to have even more criteria, or to have the membership function depend on elements from two completely different universes of discourse.

Logic Operations:

Now that we know what a statement like "X is LOW" means in fuzzy logic, how do we interpret a statement like

```
X is LOW and Y is HIGH or (not Z is MEDIUM)
```

The standard definitions in fuzzy logic are:

```
truth (not x)   = 1.0 - truth (x)
truth (x and y) = minimum (truth(x), truth(y))
truth (x or y)  = maximum (truth(x), truth(y))
```

Some researchers in fuzzy logic have explored the use of other interpretations of the AND and OR operations, but the definition for the NOT operation seems to be safe.

Note that if you plug just the values zero and one into these definitions, you get the same truth tables as you would expect from conventional Boolean logic. This is known as the EXTENSION PRINCIPLE, which states that the classical results of Boolean logic are recovered from fuzzy logic operations when all fuzzy membership grades are restricted to the traditional set {0, 1}. This effectively establishes fuzzy subsets and logic as a true generalization of classical set theory and logic. In fact, by this reasoning all crisp (traditional) subsets ARE fuzzy subsets of this very special type; and there is no conflict between fuzzy and crisp methods.

Some examples -- assume the same definition of TALL as above, and in addition, assume that we have a fuzzy subset OLD defined by the membership function:

```
old (x) = { 0,                        if age(x) < 18 yr.
            (age(x)-18 yr.)/42 yr.,   if 18 yr. <= age(x) <= 60 yr.
            1,                        if age(x) > 60 yr. }
```

And for compactness, let

```
a = X is TALL and X is OLD
b = X is TALL or X is OLD
c = not (X is TALL)
```

Then we can compute the following values.

height	age	X is TALL	X is OLD	a	b	c
3' 2"	65	0.00	1.00	0.00	1.00	1.00
5' 5"	30	0.21	0.29	0.21	0.29	0.79
5' 9"	27	0.38	0.21	0.21	0.38	0.62
5' 10"	32	0.42	0.33	0.33	0.42	0.58
6' 1"	31	0.54	0.31	0.31	0.54	0.46
7' 2"	45	1.00	0.64	0.64	1.00	0.00
3' 4"	4	0.00	0.00	0.00	0.00	1.00

For those of you who only grok the metric system, here's a dandy little conversion table:

Feet+Inches		= Meters
3'	2"	0.9652
3'	4"	1.0160

```
   5'   5"      1.6510
   5'   9"      1.7526
   5'  10"      1.7780
   6'   1"      1.8542
   7'   2"      2.1844
```

An excellent introductory article is:

Bezdek, James C, "Fuzzy Models --- What Are They, and Why?", IEEE Transactions on Fuzzy Systems, 1:1, pp. 1-6, 1993.

For more information on fuzzy logic operators, see:

Bandler, W., and Kohout, L.J., "Fuzzy Power Sets and Fuzzy Implication Operators", Fuzzy Sets and Systems 4:13-30, 1980.

Dubois, Didier, and Prade, H., "A Class of Fuzzy Measures Based on Triangle Inequalities", Int. J. Gen. Sys. 8.

The original papers on fuzzy logic include:

Zadeh, Lotfi, "Fuzzy Sets," Information and Control 8:338-353, 1965.

Zadeh, Lotfi, "Outline of a New Approach to the Analysis of Complex Systems", IEEE Trans. on Sys., Man and Cyb. 3, 1973.

Zadeh, Lotfi, "The Calculus of Fuzzy Restrictions", in Fuzzy Sets and Applications to Cognitive and Decision Making Processes, edited by L. A. Zadeh et. al., Academic Press, New York, 1975, pages 1-39.

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A brief course in Fuzzy Logic and Fuzzy Control

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Version: 1.2
Date: December 4, 1996

You may download this course from the FTP server ftp://ftp.flll.uni-linz.ac.at/pub/info/.

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Introduction to the Fuzzy Logic course

Fuzzy Logic - a powerful new technology

Fuzzy Logic has emerged as a a profitable tool for the controlling of subway systems and complex industrial processes, as well as for household and entertainment electronics, diagnosis systems and other expert systems. Although, Fuzzy Logic was invented in the United States the rapid growth of this technology has started from Japan and has now again reached the USA and Europe also.
Fuzzy Logic is still booming in Japan, the number of letters patent applied for increases exponentially. The main part deals with rather simple applications of Fuzzy Control.

Fuzzy has become a key-word for marketing. Electronic articles without Fuzzy-component gradually turn out to be dead stock. As a gag, that shows the popularity of Fuzzy Logic, there even exists a toiletpaper with "Fuzzy Logic" printed on it.
In Japan Fuzzy-research is widely supported with a huge budget. In Europe and the USA efforts are being made to catch up with the tremendous japanese success. For instance, the NASA space agency is engaged in applying Fuzzy Logic for complex docking-maneuvers.

Fuzzy Logic is basically a multivalued logic that allows intermediate values to be defined between conventional evaluations like *yes/no, true/false, black/white,* etc. Notions like *rather warm* or *pretty cold* can be formulated mathematically and processed by computers. In this way an attempt is made to apply a more human-like way of thinking in the programming of computers.
Fuzzy Logic was initiated in 1965 by **Lotfi A. Zadeh**, professor for computer science at the University of California in Berkeley.

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What is a Fuzzy Set?

The very basic notion of fuzzy systems is a *Fuzzy (sub)set.*

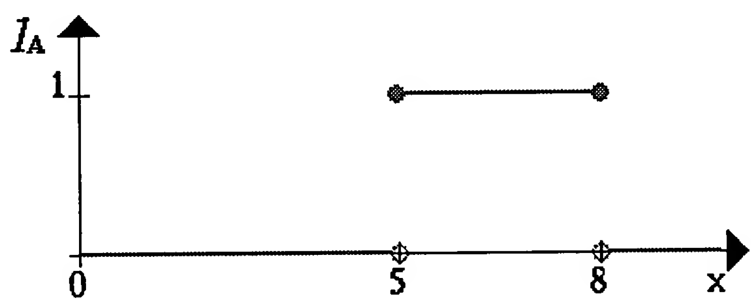
In classical mathematics we are familiar with what we call *crisp sets.*

Here is an example:

First we consider a set X of all real numbers between 0 and 10 which we call the universe of discourse. Now, let's define a subset A of X of all real-numbers in the range between 5 and 8.

```
A = [5,8]
```

We now show the set A by its characteristic function, i.e. this function assigns a number 1 or 0 to each element in X, depending on whether the element is in the subset A or not. This results in the following figure:



We can interpret the elements which have assigned the number 1 as *The elements are in the set A* and the elements which have assigned the number 0 as *The elements are not in the set A.*

This concept is sufficient for many areas of applications. But we can easily find situations where it lacks in flexibility. In order to show this consider the following example on the next page:

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What is a Fuzzy Set?

(Continued)

In this example we want to describe the set of young people. More formally we can denote

B = {set of young people}

Since - in general - age starts at 0 the lower range of this set ought to be clear. The upper range, on the other hand, is rather hard to define. As a first attempt we set the upper range to, say, 20 years. Therefore we get *B* as a crisp interval, namely:

B = [0,20]

Now the question arises: why is somebody on his 20th birthday *young* and right on the next day *not young*? Obviously, this is a structural problem, for if we move the upper bound of the range from 20 to an arbitrary point we can pose the same question.

A more natural way to construct the set *B* would be to relax the strict separation between *young* and *not young*. We will do this by allowing not only the (crisp) decision *YES he/she is in the set of young people* or *NO he/she is not in the set of young people* but more flexible phrases like *Well, he/she belongs a little bit more to the set of young people* or *NO, he/she belongs nearly not to the set of young people.*

The next page shows how a fuzzy set allows us to define such a notion as *s/he is a little young.*

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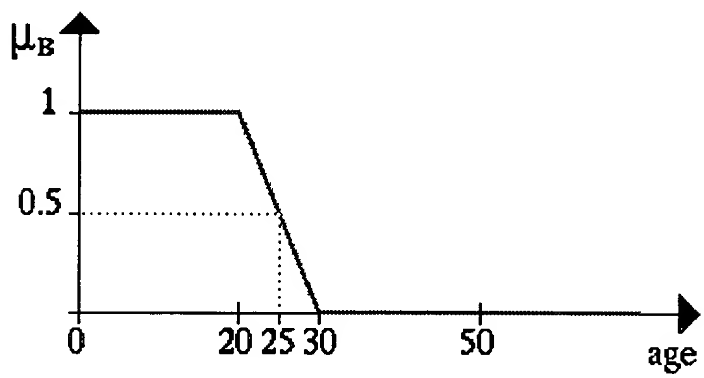
What is a Fuzzy Set?

(Continued)

As stated in the introduction we want to use fuzzy sets to make computers smarter, we now have to code the above idea more formally. In our first example we coded all the elements of the Universe of Discourse with 0 or 1. A straight way to generalize this concept is to allow more values between 0 and 1. In fact, we even allow infinite many alternatives between 0 and 1, namely the unit interval $I = [0, 1]$.

The interpretation of the numbers now assigned to all elements of the Universe of Discourse is much more difficult. Of course, again the number 1 assigned to an element means that the element is in the set *B* and 0 means that the element is definitely not in the set *B*. All other values mean a gradual membership to the set *B*.

To be more concrete we now show the set of young people similar to our first example graphically by its characteristic function.



This way a 25 years old would still be *young* to a **degree of 50 percent**.

Now you know what a **fuzzy set** is. But what can you do with it? Continue to the next page !

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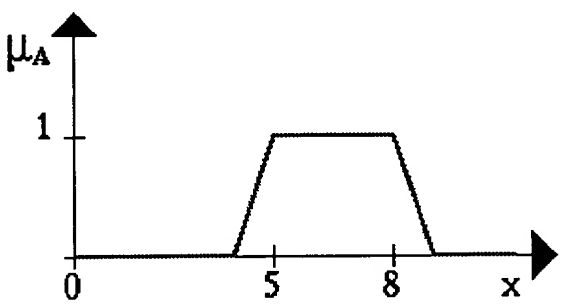
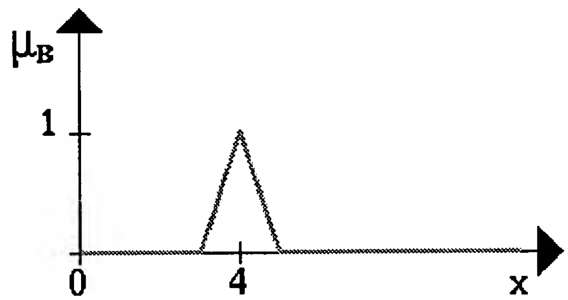
Operations on Fuzzy Sets

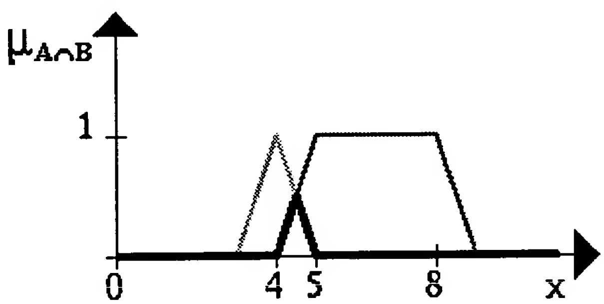
Now that we have an idea of what fuzzy sets are, we can introduce basic operations on fuzzy sets. Similar to the operations on crisp sets we also want to *intersect*, *unify* and *negate* fuzzy sets. In his very first paper about fuzzy sets, **L. A. Zadeh** suggested the **minimum operator** for the **intersection** and the **maximum operator** for the **union** of two fuzzy sets. It is easy to see that these operators coincide with the crisp unification, and intersection if we only consider the membership degrees 0 and 1.

In order to clarify this, we give a few examples. Let *A* be a fuzzy interval *between* 5 *and* 8 and *B* be a fuzzy number *about* 4. The corresponding figures are shown below.

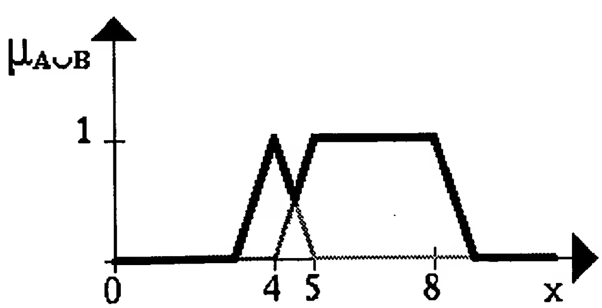




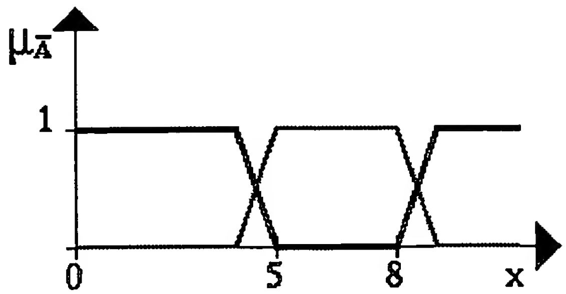
The following figure shows the fuzzy set *between* 5 *and* 8 **AND** *about* 4 (notice the blue line).



The Fuzzy set *between* 5 *and* 8 **OR** *about* 4 is shown in the next figure (again, it is the blue line).



This figure gives an example for a negation. The blue line is the **NEGATION** of the fuzzy set *A*.



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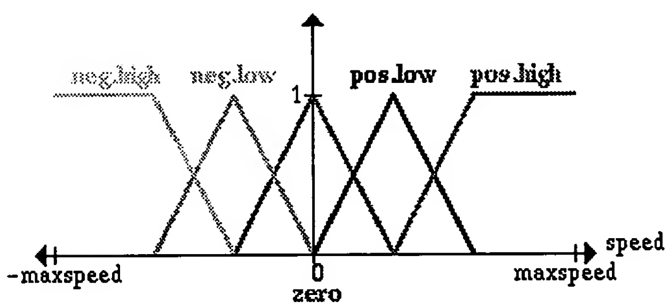
Fuzzy Control

Fuzzy controllers are the most important applications of fuzzy theory. They work rather different than conventional controllers; expert knowledge is used instead of differential equations to describe a system. This knowledge can be expressed in a very natural way using *linguistic variables*, which are described by fuzzy sets.

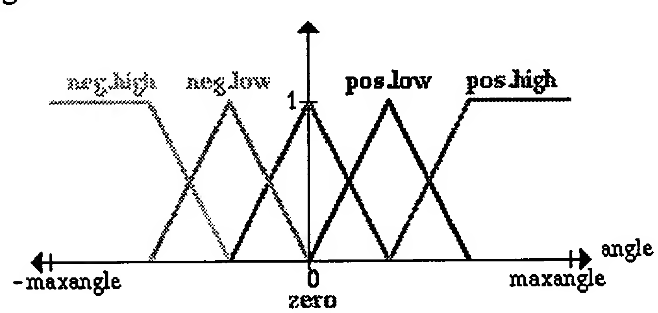
Example: Inverted pendulum

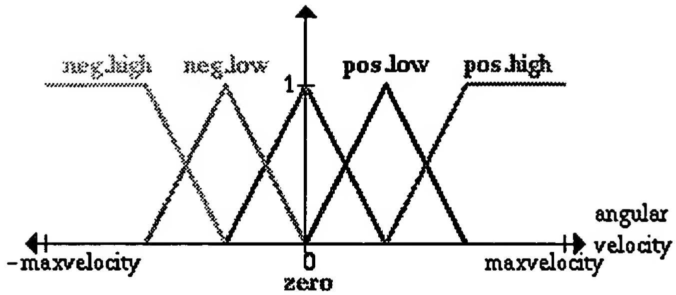
The problem is to balance a pole on a mobile platform that can move in only two directions, to the left or to the right.
First of all, we have to define (subjectively) what *high* speed, *low* speed etc. of the platform is; this is done by specifying the membership functions for the fuzzy_sets

- negative high (cyan)
- negative low (green)
- zero (red)
- positive low (blue)
- positive high (magenta)



The same is done for the angle between the platform and the pendulum and the angular velocity of this angle:





Please notice that, to make it easier, we assume that in the beginning the pole is in a *nearly upright* position so that an angle greater than, say, 45 degrees in any direction can - by definition - never occur.

On the next page we will set up some *rules* that we wish to apply in certain situations.

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Fuzzy Control

(continued)

Now we give several *rules* that say what to do in certain situations:

Consider for example that the pole is in the upright position (angle is zero) and it does not move (angular velocity is zero). Obviously this is the desired situation, and therefore we don't have to do anything (speed is zero).
Let's consider another case: the pole is in upright position as before but is in motion at *low* velocity in *positive* direction. Naturally we would have to compensate the pole's movement by moving the platform in the same direction at *low* speed.

So far we've made up two rules that can be put into a more formalized form like this:

- `If angle is zero and angular velocity is zero then speed shall be zero.`
- `If angle is zero and angular velocity is pos. low then speed shall be pos.`

We can summarize all applicable rules in a table:

				angle		
	speed	NH	NL	Z	PL	PH
v	NH			NH		
e	NL			NL	Z	
l	Z	NH	NL	Z	PL	PH
o	PL		Z	PL		
c	PH			PH		

where NH is a (usual) abbreviation for negative high, NL for negative low etc.

On the next pages we will show how these rules can be applied with concrete values for *angle* and *angular velocity*.

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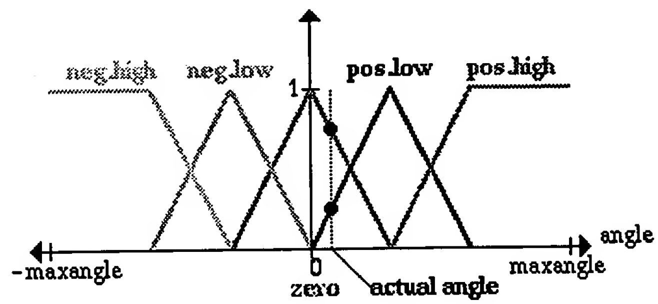
Fuzzy Control

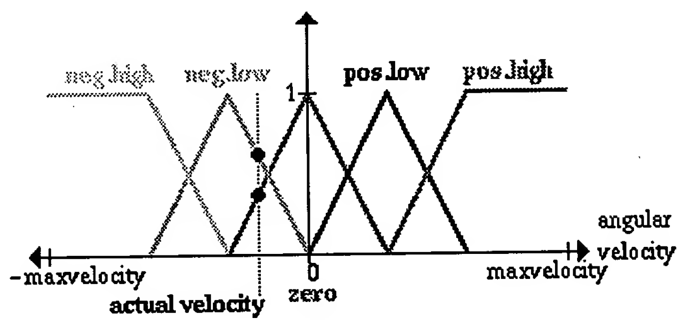
(continued)

We are going to define two explicit values for *angle* and *angular velocity* to calculate with. Consider the following situation:

An actual value for *angle:*



An actual value for *angular velocity:*



On the next page you will be able to watch how we apply our rules to this actual situation.

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Fuzzy Control

(continued)

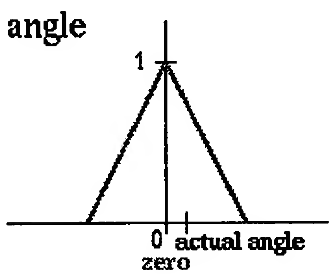
Let's apply the rule

```
If angle is zero and angular velocity is zero then speed is zero
```

to the values that we've selected on the previous page:

Click on the symbol to see how the result develops:

[Detail]



This is the linguistic variable "angle" where we zoom in on the fuzzy set "zero" and the actual angle.

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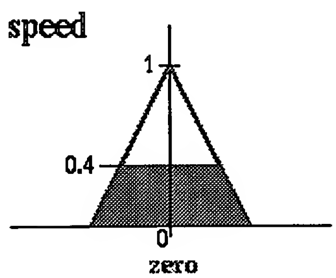
Fuzzy Control

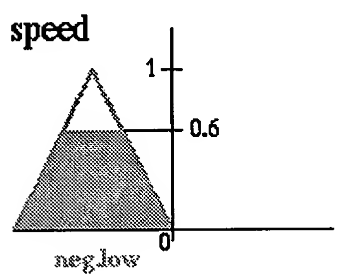
(continued)

Only four rules yield a result (they *fire*), and we overlap them into one single result.

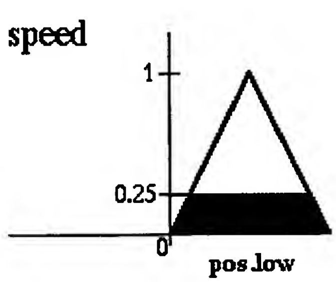
As shown on the previous page the result yielded by the rule
`if angle is zero and angular velocity is zero then speed is zero`
is:



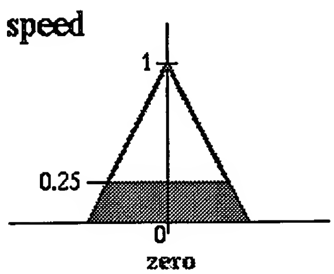
The result yielded by the rule
`if angle is zero and angular velocity is negative low then speed is negative low`
is:



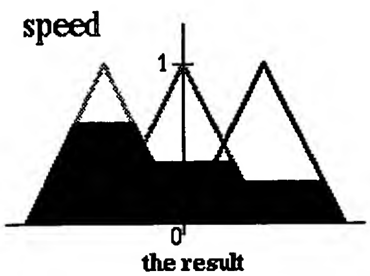
The result yielded by the rule
`if angle is positive low and angular velocity is zero then speed is positive low`
is:



The result yielded by the rule
`if angle is positive low and angular velocity is negative low then speed is zero`
is:



These four results overlapped yield the overall result:



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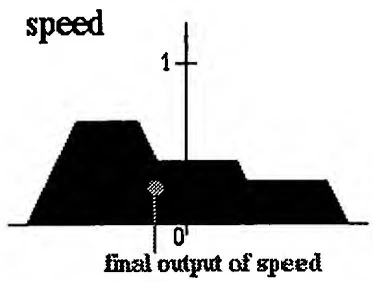
Fuzzy Control

(continued)

The result of the fuzzy controller so far is a fuzzy set (of speed), so we have to choose one representative value as the final output. There are several heuristic methods (*defuzzification methods*), one of them is e.g. to take the center of gravity of the fuzzy set:



The whole procedure is called *Mamdani* controller.

The next page deals with **applications of Fuzzy Logic.**

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Applications of Fuzzy Logic

First, we shall look at the fitness of Fuzzy Control in general terms.

The employment of Fuzzy Control is commendable...

- for very complex processes, when there is no simple mathematical model
- for highly nonlinear processes
- if the processing of (linguistically formulated) expert knowledge is to be performed

The employment of Fuzzy Control is no good idea if...

- conventional control theory yields a satisfying result
- an easily solvable and adequate mathematical model already exists
- the problem is not solvable

Now let's look at some examples where Fuzzy Control actually has been applied.

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Applications for Fuzzy Logic

(Continued)

Here are some examples of how Fuzzy Logic has been applied in reality:

- Automatic control of dam gates for **hydroelectric-powerplants**
  (Tokio Electric Pow.)
- Simplified control of **robots**
  (Hirota, Fuji Electric, Toshiba, Omron)
- **Camera aiming** for the telecast of sporting events
  (Omron)
- Substitution of an expert for the **assessment of stock exchange activities**
  (Yamaichi, Hitachi)
- Preventing unwanted temperature fluctuations in **air-conditioning systems**
  (Mitsubishi, Sharp)
- Efficient and stable control of **car-engines**
  (Nissan)
- Cruise-control for **automobiles**
  (Nissan, Subaru)
- Improved efficiency and optimized function of **industrial control applications**
  (Aptronix, Omron, Meiden, Sha, Micom, Mitsubishi, Nisshin-Denki, Oku-Electronics)
- Positioning of wafer-steppers in the **production of semiconductors**
  (Canon)
- Optimized planning of **bus time-tables**
  (Toshiba, Nippon-System, Keihan-Express)
- Archiving system for **documents**
  (Mitsubishi Elec.)
- **Prediction system** for early recognition of earthquakes
  (Inst. of Seismology Bureau of Metrology, Japan)
- **Medicine technology:** cancer diagnosis
  (Kawasaki Medical School)
- Combination of Fuzzy Logic and **Neural Nets**
  (Matsushita)
- Recognition of handwritten symbols with **pocket computers**
  (Sony)
- Recognition of motives in pictures with **video cameras**
  (Canon, Minolta)
- Automatic motor-control for **vacuum cleaners** with recognition of surface condition and degree of soiling
  (Matsushita)
- Back light control for **camcorders**
  (Sanyo)
- Compensation against vibrations in **camcorders**
  (Matsushita)
- Single button control for **washing-machines**
  (Matsushita, Hitatchi)

- **Recognition** of handwriting, objects, voice
  (CSK, Hitachi, Hosai Univ., Ricoh)
- Flight aid for **helicopters**
  (Sugeno)
- Simulation for **legal proceedings**
  (Meihi Gakuin Univ, Nagoy Univ.)
- **Software-design** for industrial processes
  (Aptronix, Harima, Ishikawajima-OC Engeneering)
- Controlling of machinery speed and temperature for **steel-works**
  (Kawasaki Steel, New-Nippon Steel, NKK)
- Controlling of **subway systems** in order to improve driving comfort, precision of halting and power economy
  (Hitachi)
- Improved fuel-consumption for **automobiles**
  (NOK, Nippon Denki Tools)
- Improved sensitiveness and efficiency for **elevator control**
  (Fujitec, Hitachi, Toshiba)
- Improved savety for **nuklear reactors**
  (Hitachi, Bernard, Nuclear Fuel div.)

Here are some projects that were developed at our lab.

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Conclusion of the Fuzzy Logic Course

This concludes our brief course in Fuzzy Logic and Fuzzy Control so far. We hope that you enjoyed it and that the explanations were of some help to you.

Stay tuned to this website for there will probably be more information on **Fuzzy Logic** in the future. (Last update: December 1996).

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